Third-order blast wave theory and its application to hypersonic flow past blunt-nosed cylinders

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The inviscid flow behind a cylindrical blast wave and its analogy with hypersonic flow past blunt-nosed cylinders is considered. Sakurai (1953, 1954) obtained a solution for the flow field behind a propagating blast wave by expanding the flow variables in power series of $1/M^2$, where M is the blast wave Mach number, and determining the coefficients of the first two terms in the series. Here the work is extended to include third-order terms. Third-order theory is shown to improve the prediction of shock wave shapes and surface pressure distributions on hemisphere-cylinder configurations at $M_\infty = 7{\cdot}7$ and $17{\cdot}18$.

1. Introduction

The purpose of this development is to extend Sakurai's solution (1953, 1954) for the unsteady flow field behind a propagating cylindrical blast wave to improve its accuracy at later times after the initial explosion. The case treated is that of a blast wave produced by an infinite line charge of constant energy per unit length.

Blast wave theory as developed by Taylor (1950) for spherical waves, Lin (1954) for cylindrical waves, and Sedov (1946) for spherical, cylindrical, and plane waves is applicable only when the blast wave is strong, i.e. for $\frac{1}{2}(\gamma-1)M^2 \gg 1$. Mel'nikova (1954) and Sakurai improved these solutions by expanding the flow variables in power series in $1/M^2$ of the form $\sum_{n=0}^{\infty} f^{(n)}(r/R)\, M^{-2n}$ where the $f^{(n)}$ are functions only of radial distance r non-dimensionalized with respect to the shock radius R as illustrated in figure 1. Sakurai obtained the series coefficients for $n = 0, 1$. The following development extends his work by obtaining the coefficients for $n = 2$, thus determining the third-order terms in the series expansions.

The analogy between steady hypersonic flow about a slender body with a corresponding unsteady flow in one less space dimension was first pointed out by Hayes (1947). This analogy applies between the constant energy flow behind a propagating cylindrical blast wave and the steady flow between the shock and outer edge of the entropy layer (Sychev 1960) on a blunt-nosed cylinder configuration.

The analogy may be illustrated as follows. Consider the blast wave produced by the explosion in a uniform atmosphere of an infinite line charge having finite constant energy per unit length. At some time after the explosion, the cylindrical wave will have a radius R and a propagation velocity $U = dR/dt$. Since the phenomena possesses cylindrical symmetry, the flow in all meridian planes is identical and is characterized by a certain radial velocity, pressure, and density field. Further, consider the flow about a blunt-nosed slender body in high-speed flight (figure 2). For any transverse plane aft of the nose where the horizontal velocity component is approximately U_∞, the following relation holds

$$M_\infty \frac{dR}{dz} = \frac{U_\infty}{c}\frac{dR}{dt}\frac{dt}{dz} = \frac{1}{c}\frac{dR}{dt}, \tag{1}$$

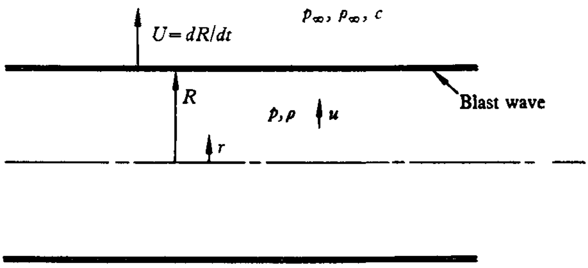

FIGURE 1. Cylindrical blast wave phenomena.

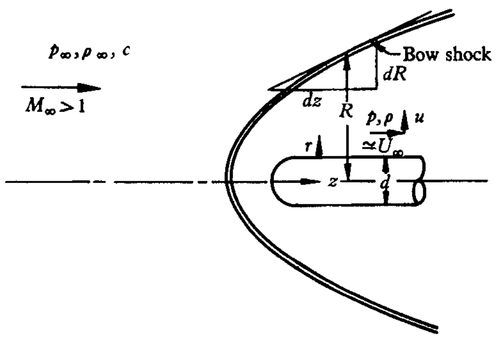

FIGURE 2. Flow about a blunt-nosed cylinder in steady flight.

where c is the speed of sound in the undisturbed stream. dR/dt, however, is analogous to the blast wave propagation velocity. Hence, (1) may be written as

$$M_\infty \frac{dR}{dz} = \frac{U}{c} = M. \tag{2}$$

Thus, if the radial velocity, pressure, and density behind the propagating blast wave are known functions of M and r/R, they are known through the above analogy in any given transverse plane aft of the nose of a blunt body whose flight Mach number and shock shape are known.

2. Mathematical analysis

Sakurai (1953, 1954) wrote the usual conservation equations of fluid mechanics in terms of the independent variables

$$x = \frac{r}{R}, \quad y = \left(\frac{c}{U}\right)^2, \tag{3}$$

and substituted into the differential equations and shock boundary conditions the following assumed power series expansions for particle velocity, pressure, and density.

$$u = U[f^{(0)}(x) + f^{(1)}(x)\,y + f^{(2)}(x)\,y^2 + \dots], \tag{4}$$

$$p = p_\infty y^{-1}[g^{(0)}(x) + g^{(1)}(x)\,y + g^{(2)}(x)\,y^2 + \dots], \tag{5}$$

$$\rho = \rho_\infty[h^{(0)}(x) + h^{(1)}(x)\,y + h^{(2)}(x)\,y^2 + \dots]. \tag{6}$$

x	$\phi^{(2)}$	$\psi^{(2)}$	$\chi^{(2)}$
1·0	0	0	25
0·98	0·4229	−1·6187	0·4992
0·96	0·5285	−1·1517	−8·0613
0·94	0·5481	−0·2062	−9·4892
0·92	0·5400	0·6638	−8·3281
0·90	0·5144	1·3174	−6·1815
0·88	0·4680	1·7416	−3·6955
0·86	0·3917	1·9716	−1·3643
0·84	0·3053	2·0733	0·3793
0·82	0·2079	2·0837	2·2534
0·80	0·1031	2·0339	3·5950
0·78	−0·0049	1·952	4·8404
0·76	−0·1112	1·8527	5·6202
0·74	−0·2211	1·7468	6·6244
0·72	−0·3271	1·6422	7·1700
0·70	−0·4328	1·5441	7·8143
0·68	−0·5419	1·4549	8·1487
0·66	−0·6510	1·3759	8·5018
0·64	−0·7569	1·3317	8·8549
0·62	−0·8603	1·2469	9·2012
0·60	−0·9630	1·1963	9·4030
0·58	−1·0692	1·1519	9·9539
0·56	−1·1796	1·1137	10·1068
0·54	−1·2907	1·0815	10·5517
0·52	−1·4064	1·0551	10·6582
0·50	−0·5290	1·0342	10·7647
0·48	−1·6600	1·0174	—
0·46	−1·8008	1·0043	—
0·44	−1·9532	0·9937	—
0·42	−2·1231	0·9856	—
0·40	−2·3282	0·9802	10·75
0·30	—	0·9633	—
0·20	—	0·9571	—
0·10	—	0·9546	—
0·00	—	0·9478	—

TABLE 1. Numerical solutions for $\phi^{(2)}$, $\psi^{(2)}$, $\chi^{(2)}$

The equation of like powers of y then results in an infinite number of sets of equations for the series coefficients which may be solved successively. Further substitution of (4), (5), and (6) into the mathematical expression of equality of energy per unit length of the flow contained within the cylindrical shock and that of the original line charge results in the following expression for $R(y)$

$$y(R_0/R)^2 = J_0(1 + \lambda_1 y + \tfrac{1}{2}\lambda_2 y^2 + \dots), \tag{7}$$

where $R_0^2 = E/p_\infty$, E being the energy per unit length of the original line charge, and J_0 and the λ_i's are constants.

The object of this work is to determine $f^{(2)}$, $g^{(2)}$, $h^{(2)}$, and λ_2 by solution of the set of equations resulting from equation of like powers of y^2. We define

$$f^{(2)}(x) = \{x - f^{(0)}(x)\}\,\phi^{(2)}(x), \tag{8}$$

$$g^{(2)}(x) = g^{(0)}(x)\,\psi^{(2)}(x), \tag{9}$$

$$h^{(2)}(x) = h^{(0)}(x)\,\chi^{(2)}(x), \tag{10}$$

and further, since $\phi^{(2)}$, $\psi^{(2)}$, and $\chi^{(2)}$ depend on λ_2, which cannot be obtained until $\phi^{(2)}$, $\psi^{(2)}$, and $\chi^{(2)}$ are known, we let

$$\phi^{(2)} = \phi_1^{(2)} + \lambda_2 \phi_2^{(2)}, \tag{11}$$

$$\psi^{(2)} = \psi_1^{(2)} + \lambda_2 \psi_2^{(2)}, \tag{12}$$

$$\chi^{(2)} = \chi_1^{(2)} + \lambda_2 \chi_2^{(2)}. \tag{13}$$

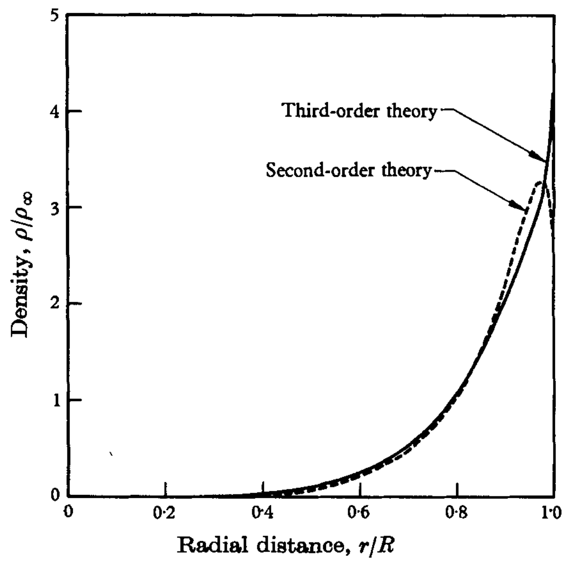

FIGURE 3. Density distribution comparison; $M = 3$, $\gamma = 1{\cdot}4$.

This leads, after some manipulation, to two pairs of coupled linear ordinary first-order differential equations for $\phi_1^{(2)}$, $\psi_1^{(2)}$, and $\phi_2^{(2)}$, $\psi_2^{(2)}$ whose coefficients derive from the previous approximations, together with appropriate boundary conditions. For details see Swigart (1960). These equations were integrated on a Rand 1103A computer utilizing the Runge–Kutta method. The results yield $\lambda_2 = 2{\cdot}7373$. The values of $\phi^{(2)}$, $\psi^{(2)}$, and $\chi^{(2)}$ then determined are given in table 1.

3. Blast wave results

Non-dimensionalized radial velocity, pressure, and density accurate to the third-order were obtained using equations (4)–(6) and the first- and second-order results given in Sakurai (1953, 1954). A typical result is given in figure 3 in which density distributions given by second- and third-order theory at a shock Mach number of 3 are compared. Note that the physically incorrect maximum in the second-order curve is corrected by the third-order term. Further results indicate that the third-order term in the density expansion is negligible for M above 5, whereas terms higher than third-order may not be neglected for M less than 3. Corresponding results hold for radial velocity and pressure.

4. Blunt-body shock-wave shapes and surface-pressure distributions

As previously mentioned, an analogy exists between the constant-energy non-steady similar flow behind the blast wave and steady hypersonic flow about a blunt-nosed cylinder at zero incidence. For this case, the energy per unit length of the original line charge in the blast wave problem is identified with the nose drag, D, of the axisymmetric body.

Equation (7) is a differential equation relating the speed of the shock front $U = dR/dt$ with time t. Solution of this equation yields $R = R(t;\, E)$. Substitution of z/U_∞ for t and $(D/2\pi p_\infty)^{\frac{1}{2}}$ for R_0 yields an equation for the shock shape about a blunt-nosed slender body as a function of axial distance from the body nose and flight Mach number. The resulting third-order equation is

$$\frac{R}{d} = 0{\cdot}7951\,(C_D)^{\frac{1}{4}}\left(\frac{z}{d}\right)^{\frac{1}{2}}\left[1+\frac{1{\cdot}5730}{M_\infty^2\,(C_D)^{\frac{1}{2}}}\frac{z}{d}-\frac{5{\cdot}8037}{M_\infty^4\,C_D}\left(\frac{z}{d}\right)^2\right], \tag{14}$$

where C_D is the nose drag coefficient. Note that (14) is more general than the corresponding second-order equation reported by Lees & Kubota (1957) in that it depends parametrically on body nose-drag coefficient. Lees & Kubota's equation

$$\frac{R}{d} = 0{\cdot}78\left(\frac{z}{d}\right)^{\frac{1}{2}}\left(1+\frac{1{\cdot}62}{M_\infty^2}\frac{z}{d}\right) \tag{15}$$

has embodied in its derivation a nose-drag coefficient for a hemisphere at $M_\infty = 7{\cdot}7$ as determined by modified Newtonian theory

$$C_D = \frac{1}{\gamma M_\infty^2}\left(\frac{p_m}{p_\infty}+1\right), \tag{16}$$

where p_m is the pressure at the stagnation point.

For all Mach numbers above and slightly below 7·7, the neglect of the nose-drag variation with Mach number is justifiable. For nose geometries that differ considerably from a hemisphere, a value of C_D corresponding to the geometry should be used rather than (16). When the value for C_D obtained using (16) at $M_\infty = 7{\cdot}7$ is introduced in (14), the first two terms are identically equation (15).

A comparison with experimental data of the shock-wave shapes obtained using two and three terms of (14) is given in figure 4 for a hemisphere-cylinder at $M_\infty = 7{\cdot}7$. Although the second-order curve lies closer to the experimental points over the entire body length, third-order theory yields a more nearly parallel curve.

Differentiation of the third-order solution to (7) results in $U(t;E) = dR(t;E)/dt$. Substitution of this expression into (5) evaluated at $x = 0$ and use of the analogy equalities results in the following third-order expression for surface-pressure distribution

$$\frac{P_b}{P_\infty} = 0{\cdot}0687\frac{M_\infty^2 (C_D)^{\frac{1}{2}}}{z/d} + 0{\cdot}405 + 0{\cdot}150\frac{z/d}{M_\infty^2 (C_D)^{\frac{1}{2}}}. \tag{17}$$

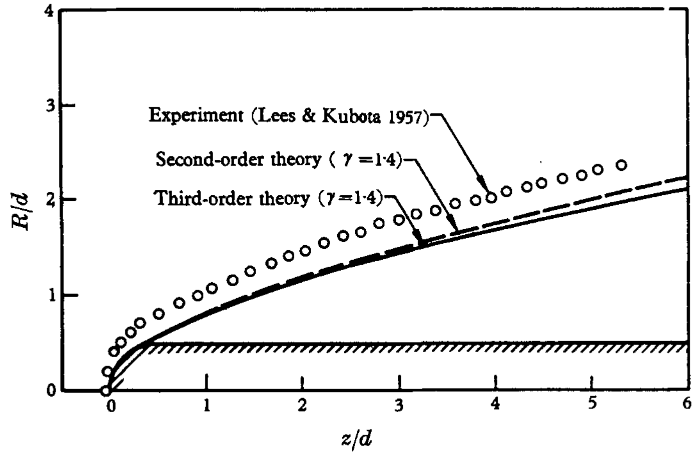

FIGURE 4. Shock shape about a hemisphere-cylinder; $M_\infty = 7{\cdot}7$.

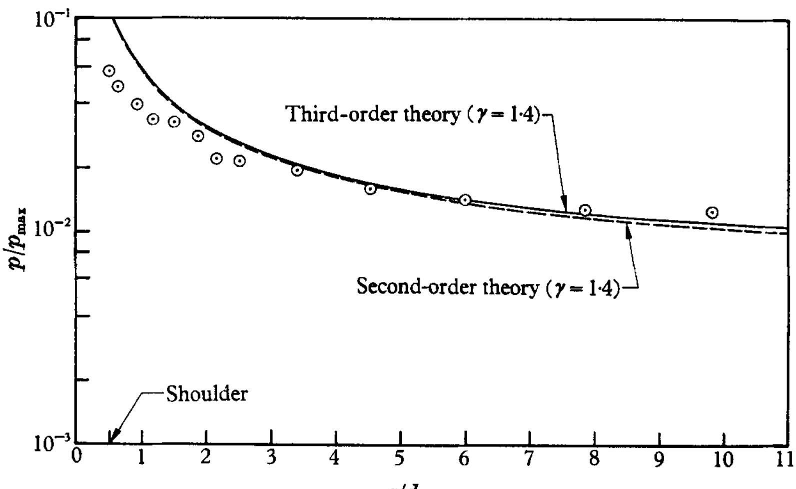

FIGURE 5. Surface pressure distribution on a hemisphere-cylinder; $M_\infty = 7{\cdot}7$.
⊙ Experiment (Lees & Kubota 1957).

The same comments regarding the nose-drag coefficient in (14) apply to (17) and the corresponding second-order expression for surface pressure distribution reported in Lees & Kubota (1957).

Comparisons of the results obtained using two and three terms of (17) with experimental data at $M_\infty = 7{\cdot}7$ (Lees & Kubota 1957) and with a numerical solution at $M_\infty = 17{\cdot}98$ (Feldman 1959) are given in figures 5 and 6. Note that the correction to the second-order curve increases with increasing z/d.

The significant contribution of third-order theory in both cases is, as with shock shape, the more accurate prediction of the slope of the pressure-distribution curve rather than correction of the value of the pressure at a given z/d. Indeed, depending on the z/d under consideration, third-order theory may be either more or less accurate than second-order theory in predicting the value of the pressure.

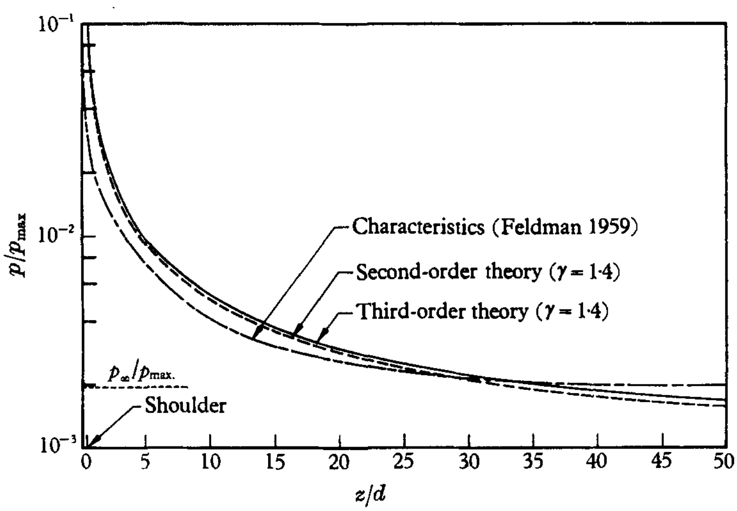

FIGURE 6. Surface pressure distribution on a hemisphere-cylinder; $M_\infty = 17{\cdot}98$.

Hence, since the predicted shoulder pressure is incorrect and changes very little between second- and third-order theories, application of one of the curve-shifting schemes to correct this deficiency (Casaccio 1959) will result in a more accurate pressure-distribution prediction by third-order theory than by second-order theory for the entire lengths of the bodies considered.

We might again point out that, due to the large entropy gradient in the vicinity of the body, the blast-wave analogy is not valid in this region. The pressure distribution obtained by the analogy actually applies on a body derived by taking the entropy layer into account (Sychev 1960). This fact accounts for some of the discrepancy between experimental or exact numerical results and blast wave theory.

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